



## Soutenance de thèse de Gilles BAROIN

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**Title :** Applications of graph theory to musical objects. Modeling, visualization in hyperspace

**Key words :** Symmetry, Mathematics, Music, Hypersphere, Pitch Space, Chordal Space, Tonnetz, Spectral Projection, Graph Theory, Ideograms, Animated CGI, Quaternions. Planet-4D, Computational Musicology, Mathemusical, Set Theory.

**Abstract :**

At the frontier between Music and Mathematics, this study presents an original geometrical musical space used for musical analysis and pedagogy. Using different schemes, mathematicians and music theorists have demonstrated that the tempered twelve tones pitch space can be considered as a combination of minor and major thirds. We use the Cartesian product of two circular graphs  $C_3 \square C_4$  to build the Planet Graph that matches this concept. Since the decomposition involves two sets and each pitch class being a unique combination of these two sub-groups, we use a graph coloration based on complex numbers and introduce the concept of bi-dimensional ideograms. We perform a spectral analysis of the Planet Graph to determine its Eigen spaces and obtain geometrical coordinates. The resulting model, called Planet-4D, grants each symbol an equivalent physical position, and involves more symmetries than any discrete 3D model. From there, we build a four dimensional chordal space where perfect chords lie on a hypersphere. We finally extend this concept to display any set of pitches in an atonal context. In the second section we construct the graphs of some existing musical objects such as keyboards, tone networks (Tonnetze), chordal spaces or modulation schemes. We apply spectral projections to visualize the symmetries that are inherent to these objects. This work concludes with musical studies of tonal and atonal pieces, performed with the help of the visualization tools designed in this study.